Comparison of the Specific Resistances of Cakes Formed in Filters and Centrifuges

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Resistances to the flow of fluids through porous beds were determined in a compression-permeability cell, a vacuum test filter, and an experimental centrifugal filter. The study was made on nearly incompressible beds composed of Lucite spheres having a given size distribution. The results show that liquid flow through the cakes in all three kinds of equipment can be described by a common basic equation, the terms of which are modified to reflect the type of operation. The centrifugal-drainage data confirm the form of the centrifugal-filtration equation. The specific resistances deduced from vacuum-filtration and centrifugal-drainage tests agree with permeability-cell alphas within 16% at the same ΔP and within 2 to 3% at the same porosity.

The development of the principles and the experimental techniques for investigating filtration and centrifugal filtration in the last decade has encouraged studies of the two separate operations as a single one. Storrow and his co-workers (5) showed experimentally that the same type of equations applies to both types of filtration. Grace (4) demonstrated in several cases that there was a reasonably close agreement between the specific resistance measured in filtration and that measured in centrifugation apparatus. The objective of this work was to study further the degree of agreement which could be obtained between the specific resistances measured in permeability cells, in vacuum filters, and in a centrifuge. Also the authors wished to compare their results with those predicted from the Kozeny-Carman equation.

Centrifugal filtration is basically an operation in which a centrifugal-force field is substituted for the pressure-driving force encountered in normal pressure or vacuum filtration. There are, however, a number of differences, and these have been pointed out by workers in the field who have proposed equations to describe the operation.

Maloney (8) in the first of his annual reviews of centrifugation suggested that an analytical relationship for centrifugal filtration could be developed by extending the filtration equations to the centrifugal operation. Starting with the basic differential equation

$$\frac{dV}{d\theta} = \frac{A\,\Delta P\, g_c}{\mu\left[\dfrac{\alpha w V}{A} + R_m\right]} = \frac{A\,\Delta P\, g_c}{\mu\left[\dfrac{\alpha_v c V}{A} + R_m\right]} \tag{1}$$

and neglecting the resistance associated with the filter septum, he proposed essentially this expression:

$$\frac{dV}{d\theta} = \frac{A_{avg}^2\,(\Delta P)_L\, g_c}{\mu\,\alpha_v c V} \tag{2}$$

Storrow and co-workers (1, 5, 6, 7) developed and employed an equation which they wrote as

$$Q = \frac{K_s\, 2\pi h}{g\,\mu \ln \dfrac{r_o}{r_c}}\left[\frac{1}{2}(2\pi N)^2\,(r_o^2 - r_1^2)\right] \tag{3}$$

but which is equivalent to

$$\frac{dV}{d\theta} = \frac{A_{avg}\,A_{lm}\,(\Delta P) g_c}{\mu\, c\, V}\quad \frac{K_s}{\rho_L g} \tag{4}$$

Other than characterizing the bed by a permeability expression $K_s/\rho_L g$ instead of a specific resistance, Storrow's equation differs from Equation (2) only in the use of the product $A_{avg} A_{lm}$ instead of A_{avg}^2.

When the ratio $r_o/r_c = 2$, A_{avg} will be about 4% higher than A_{lm}, and when the ratio $r_o/r_c = 1.5$, A_{avg} is only 1.2% higher than A_{lm}. Therefore Equations (2) and (4) will give essentially the same answers for all cases except those of thick cakes in small-diameter machines.

Storrow and Haruni (6) confirmed the form of the equations and found satisfactory agreement in their tests of the Q, N^2, and $(r_o^2 - r_L^2)$ relationships. They varied the viscosity from 1 to 3 centipoises and used both 9- and 18-in. -diameter centrifuge baskets. Although these authors concluded that the cake layer next to the cloth offers a much higher resistance than the layers subsequently laid down, they did not include a separate term for the initial resistance.

Grace (4) developed rigorously the basic rate equation for centrifugal filtration. But the solution for the general case is not possible, since the point specific resistance and point porosity are not known as a function of basket radius. The average specific resistance cannot be computed directly from compression-permeability data alone as is possible for most pressure filtrations.

For the special case of a nearly incompressible cake and in the absence of kinetic-energy effects, Grace's general equation simplifies to

$$\frac{dV}{d\theta} = \frac{(\Delta P)_L\, g_c}{\mu\left[\dfrac{\alpha w V}{A_{1m}\,A_{avg}} + \dfrac{R_m}{A_o}\right]} \tag{5}$$

The equation thus takes the initial resistance into account and associates A_o with it. The area-squared factor entering the cake-resistance term is made up of the product of the log mean area and the arithmetic mean area.

Grace's equation is seen to be similar to Storrow's except for the inclusion of the initial resistance term. The only additional difference from Equation (2) is the use of the log mean area as one of the factors in the A^2 term, this point having been discussed previously.

Grace obtained data on the flow rate of water through cakes of various thicknesses deposited in a laboratory centrifuge. In some of his runs he encountered difficulties with filter cakes developing cracks.; in other runs the apparently found good agreement (within 10%) for diatomaceous earth between centrifugal- and pressure-filtration average specific resistances and with permeability-cell data. Most unexpectedly Grace also found agreement within 25% for semicompressible cakes of cellulose and carbon filter aids and of titanium dioxide powder. Grace thought that the materials and the

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method of testing may have allowed the expected discrepancies to be masked by the scatter of the experimental points. While recognizing the significant contributions made by Grace one should, because of the uncertainties mentioned, have further confirmation and comparison of the resistances of porous beds composed of spherical particles to liquid flow.

EXPERIMENTAL PROCEDURE

All the studies in this work were carried out on porous beds or filter cakes composed of Lucite spheres. Lucite 4F molding powder was used, as the particles have a remarkably uniform spherical shape which avoids the difficulties of trying to characterize a mixture of various shapes. For comparison purposes it also has the advantage of having a value of unity for the sphericity. The Lucite particles are solid and physically and chemically inert in water, neither softening nor swelling in it. Lucite 4F, however, is soluble in acetone, ethylene dichloride, and toluene, and these solvents may be used for removing embedded particles from screens and filter cloths. Owing to a specific gravity of only 1.18 the difficulty of maintaining Lucite particles in suspension prior to filtration is minimized.

A single size distribution of Lucite spheres was used in all experiments. The specific surface determined from microscopic counts was 890 sq. cm./cc. Half of the particles were below 40 μ in diameter.

The Lucite particles would not disperse in tap or distilled water. The contact angle between distilled water and a Lucite surface was measured as about 64 deg. by the shadowgraph technique. The addition of small amounts of certain wetting agents would, however, cause the particles to disperse readily. In effectiveness Dreft appeared equal to or better than any of the other surfactants tested, and a 0.05 wt. % solution of Dreft in water was used in making up the slurries from which the Lucite cakes were deposited. The 0.05% concentration was chosen because the surface-tension–concentration curve in this region was almost flat, the surface tension therefore being unaffected by small variaations in Dreft concentration. Neither the obensity nor the viscosity of water was measurably altered by Dreft at this concentration.

Description of the Permeability Cell

The point porosity and specific resistance of a cake as a function of compressive stress can be determined most conveniently in a compression-permeability cell. This device allows a mechanical stress to be placed on a porous bed independently of the fluid head. The pressure drop caused by fluid flow can be made very small compared with the compressive stress which is imposed uniformly over the solid matrix of the cake.

A diagram of the compression-permeability cell is shown in Figure 1. and is similar to the one described by Grace (*4*). The brass cylinder in which the cakes

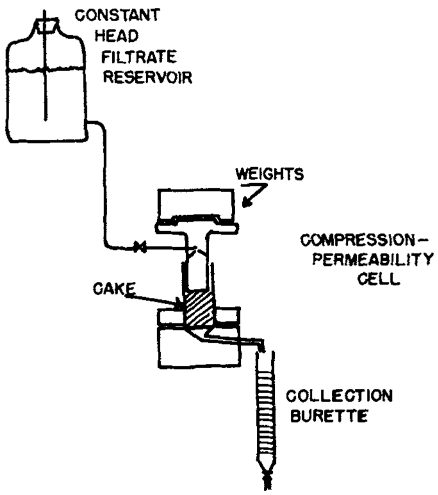

Fig. 1. Diagram of permeability-cell arrangement.

were deposited had an internal diameter of 1.133 in., and the sintered stainless steel disks between which the cake was confined were 0.185 in. thick.

In operation mechanical pressure was applied to the filter cake by loading the piston with calibrated weights. Filtrate flowed from a constant-head reservoir into the hollow piston, through the porous piston end plate and filter paper, down through the filter cake, and thence through the bottom paper and drain plate into the drainage base. Movement of the piston was measured by a dial guage clamped to the cylinder and fitted against the piston flange. The flow rate at each value of compressive stress was found by timing with a stop watch the accumulation of a volume interval of filtrate in the collection burette.

The Lucite cakes of 6.65 g. each were deposited from 10% by weight suspensions of Lucite spheres in a 0.05 wt. % Dreft solution. A vacuum of 1 in. Hg was applied under the bottom disk to increase the rate of deposition so as to minimize the effects of differential settling. Addition filtrate added to the cylinder during the deposition of the cake kept the cake surface from becoming dry. It is of utmost importance in all studies, such as the ones described in this work, that the cake never be allowed to become dry. If it does, certain portions of the cake become airbound and the specific resistance changes.

The initial thickness and volume of the porous bed were found from micrometer measurements of the distance between piston and cylinder flanges. The filtrate prior to being placed in the reservoir was filtered through the same kind of filter paper as was used to confine the cake in the cell.

The first measurements were taken with only the weight of the piston to provide the compressive stress, the value being 3.28 lb./sq. in. After the first set of measurements a solid brass cylinder of known weight was placed on top of the piston to provide the compressive stress under which the next point was to be determined. Generally all the descent of the piston occurred within several minutes after the addition of the weight to the piston, but 15 to 20 min. was allowed between each set of measurements. Micrometer measurements between the piston and cylinder flanges were made often during a run, and they were always in good agreement with heights indicated by dial-gauge readings and initial height.

Description of the Vacuum Filter

The behavior of this Lucite-water system during filtration was established through tests with a small vacuum filter. Trials were made with other kinds of test filters, both pressure and vacuum types, but operation was not so satisfactory or convenient as with the vacuum filter shown in Figure 2.

The filter cell was made up of a cell body, a screen support ring which fit into the cell and upon which the stainless screen rested, and a sleeve which formed the cake space. The inside diameter of the sleeve was 1.065 in. and its length 2-9/64 in. Other than the screen all parts were machined from brass. A short length of standard ¼-in. galvanized pipe was screwed into the bottom of the cell body and served to transfer the filtrate to the ½-in. polyethylene delivery tubing. This line was made as large as convenience would allow, thereby making line pressure drop negligible.

The test filter was immersed in a Lucite-Dreft-water slurry, and in constant ΔP tests cakes about 1½ in. in diameter were deposited by applying vacua ranging from 6.4 to 24 in. Hg. The general arrangement of the filtration-test equipment is shown in the line diagram of Figure 2.

All water used in the vacuum-filtration runs was deaerated and boiled under 28 in. Hg vacuum before the slurry was made up. Weighed amounts of Lucite and Dreft were added to give a solids concentration of about 2% in a 0.05 wt. % Dreft solution.

In operation the filter cell and the filtrate delivery line were filled with distilled water first, and then the filter cell was immersed in the slurry facing upward. During filtration simultaneous readings of filtrate level and time were taken every 10 sec. After the cake had reached a thickness of approximately 1½ in. the run was concluded by raising the test filter out of the slurry, thus draining the filtrate line and dewatering the cake.

After a run the cake-containing sleeve was removed from the filter body by gently turning the sleeve. The cake was expelled with a flat plunger from the sleeve onto a weighed watch glass. The exact concentration of the slurry was computed from the weight of the dry cake and the volume (corrected) of the filtrate. The surfaces of the filter cakes at the end of the tests were always flat, and after dewatering the cakes were quite compact. Essentially complete recovery of the cakes was obtained. The only loss of cake consisted of those solids that remained embedded in the filter cloth after it had been scraped with a spatula.

A series of runs was made as rapidly as possible to minimize the amount of air dissolving in the slurry, and the runs were made in the order of decreasing pressure differences.

Different filter cloths were used in each run of a series, but the same set of cloths was used in successive series. These cloths were cut from preshrunk cotton drill. They were washed with Dreft solution and rinsed between runs but were not treated with toluene to dissolve out embedded Lucite spheres. Satisfactorily reproducible results for the value of the filter-cloth resistance were obtained without using the toluene wash.

Centrifugal-Filtration Equipment

The experimental centrifuge used in this portion of the work was a 12-in. diameter basket mounted on a horizontal shaft and driven by a variable-speed transmission coupled to an electric motor. Mounting a basket horizontally along its axis eliminated trouble with uneven cakes or cakes of varying thickness. This is a problem with vertically mounted baskets which has been reported by many investigators.

The filter cloth was made of the same material as was used in the filtration tests, and it was tailored to fit the basket. The portions of the cloth which covered the bottom and top lip of the basket were waterproofed with a rubber cement. Tests made with the cloth both in and out of the centrifuge showed that water could flow only through the sides of the cloth. Since the area for filtration must be accurately known, no filtration is permitted except through the fabric at the outer edge of the basket. A 4 by 8-mesh calendered stainless steel backing screen which fitted closely inside the basket provided support for the cloth.

Figure 3 is a sketch of the general arrangement of the equipment. The slurry was mixed in the 10-gal. galvanized tank at the upper right. A variable-speed mixer in combination with the corrugated sides of the container (acting as baffles) kept the slurry well agitated. At the centrifuge the Lucite slurry was directed downward at an angle toward the wall of the centrifuge through a feed nozzle made from a Bunsen burner wingtip. The wingtip, which was first used by Grace (*4*) for this purpose, was effective in spreading the slurry feed into a thin, flat stream across the filtering surface. The water was passed through a cartridge filter ahead of the rotameter, and a similar wing-tip nozzle was used for feeding the water to the cake during the drainage-rate runs. Drainage of the filtrate was by gravity from the outlet at the botton of the shell surrounding the basket. The shell was made from a short length of 20-in. diameter steel pipe. Cake thicknesses and liquid levels in the basket were measured by means of a probe which was movable from the basket wall toward the center along a radius. With this device cake thickness and liquid levels were measured to, and reproducible within 1/64 in. Rotational speeds were measured by a hand tachometer at the end of the shaft which passed through the middle of the basket. These readings were checked occasionally with a Strobotac and were always in good agreement.

The centrifugal tests which were conducted may be classified into several categories, of which the centrifugal-drainage studies are the first. In these the centrifuge speed and the fluid level in the basket were held constant. Under these conditions 1-lb. increments of cake solids were deposited up to a total cake weight of 5 lb., and the flow of filtrate through each weight cake was measured. The runs therefore, are analogous to constant-pressure filtration tests except that rate measurements were made at given increments instead of cumulative measurements being taken continuously. The second kind of centrifugal tests attempted consisted of regular centrifugal filtrations conducted under true constant-head conditions, for which cumulative filtrate volume was found as a function of time. The third set of centrifuge experiments, not treated in this paper, was made primarily for measurement of residual moisture in spun or wrungout centrifugal-filter cakes.

In all centrifugal experiments the filter cloth was fitted into the basket and spun in place at the highest speed of the centrifuge, 1,800 rev./min. approximately, while water was being fed into the basket. The location of the cloth surface was checked with the probe before the start of each run. In the drainage-rate tests the tip of the probe was then set where it was desired to maintain the liquid level.

For the drainage-rate runs the flow of feed slurry into the basket for deposition of filter cake was always started with water flowing through the basket, which had been brought up to desired speed. Filtration rates were very high with thin cakes, and the controlled addition of water ensured that the cake surface was always submerged and so could not become dry. The flow rate of extra water was reduced as the cake became thicker, and in the case of the constant-head drainage-rate studies both water and slurry rates were controlled to maintain the liquid level constant at the point of the probe. As the slurry tank emptied, it was washed down to make certain that all the weighed quantity of Lucite was fed to the centrifuge. When all the solids had been deposited, the water flow rate which would just keep the water level at the point of the probe was measured. In the drainage-rate runs the next increment of cake weight would be deposited from slurry and the procedure repeated at the same speed and liquid level.

At the conclusion of a run the water was shut off and the cake spun dry. When no more liquid was being wrung out at that speed, the centrifuge was stopped. The scale was read with the probe at the cake surface, and from this reading the thickness of the cake was computed. The thickness at various points also was measured directly with a depth gauge. Agreement within 1/64-in. usually was achieved. One or more samples of the cake were then removed for moisture analysis.

In the second type of centrifugal test, a regular centrifugal-filtration operation under constant-head conditions, no additional water was fed to the centrifuge during filtration. With the centrifuge rotating at a constant speed, cake deposition took place as rapidly as possible until the desired constant slurry level could be maintained. Filtration was then completed at a fixed speed, and liquid level and corresponding values of cumulative time and cumulative filtrate volume were obtained during the filtration.

EXPERIMENTAL RESULTS

Permeability-Cell Measurements

Compressive stresses applied to the Lucite beds in the compression-permeability cell ranged from about 3 to 56 lb./sq. in. The corresponding values of porosity and point specific resistance are found in Table 1 for two runs in which the Lucite beds were the same weight but furnished slightly different

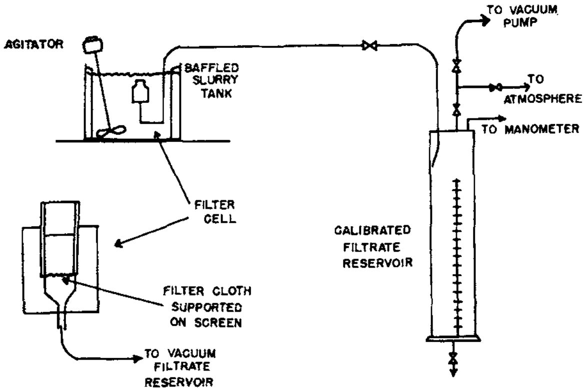

Fig. 2. Line diagram of vacuum-filter arrangement.

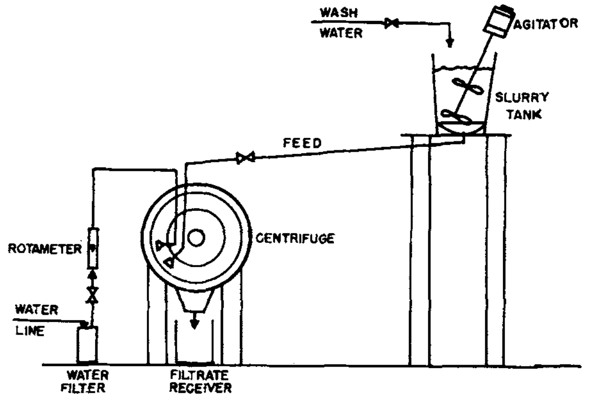

Fig. 3. Line diagram of arrangement of centrifugal filter.

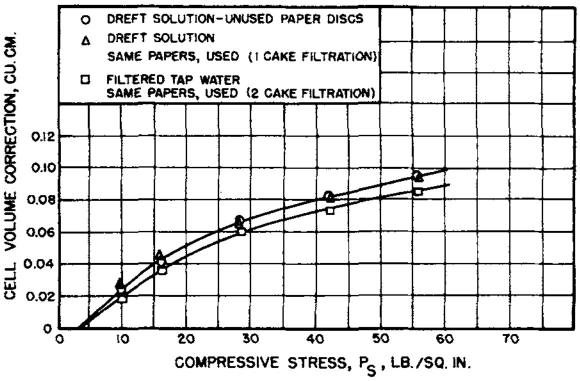

Fig. 4. Cell-volume correction factor.

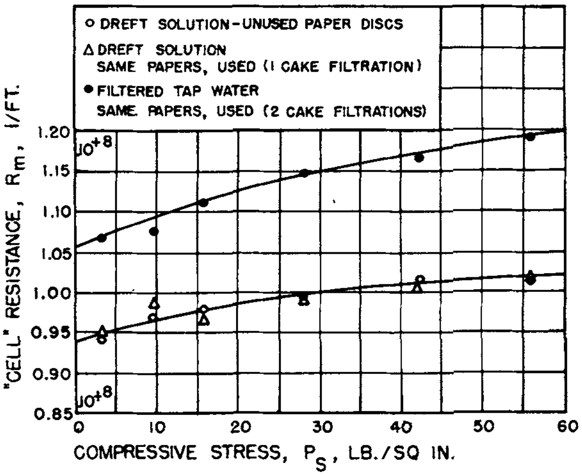

Fig. 5. Variation of cell resistance as a function of the compressive stress.

levels of porosity. A solution of 0.05 wt. % Dreft in distilled water constituted the filtrate in each of these tests. Another test was conducted however in which tap water was passed through the permeability cell containing a Lucite bed. These data are also shown in Table 1. This additional experiment was made because in the centrifugal-drainage–rate studies tap water was passed through the cakes.

Table 1 shows that the porosity and specific resistance change relatively little over the seventeen-fold range in compressive stress, thus indicating the incompressible nature of the cakes. In general the specific resistance falls in the range of 7 to 9 (10^8) ft./lb. These values are significantly lower than those usually encountered in filtration work. The specific resistance of most materials of importance in filtration is in the range of 10^{10} to 10^{13} ft./lb. The specific resistances of several commercial filter aids as reported by Grace (4) are between 10^{10} and 10^{11} ft./lb., an order of magnitude greater than for the Lucite beds.

The initial or cell resistance for the permeability tests was determined from blank cell runs, and from the results the cell-volume correction factors of Figure 4 also were derived. Each of the blank cell runs was made with the same two disks of filter paper inserted between the sintered stainless plates. CC-12 was made with 0.05 % Dreft solution and the previously unused filter papers. The filter papers were then used in the cake-permeability test CC-13, after which they were subjected to another blank-cell test CC-14 to learn how their filtration characteristics might have been changed. The papers were washed only with distilled water after the cake-permeability run. As the curve of Figure 5 shows, the cell resistance was found to be almost identical for these two tests. The same two filter papers then were used in run CC-16 with tap water to establish the cell-resistance values for run CC-17 to follow. As noted from Figure 5 the resistance to flow of tap water is 10 to 15 % greater than for Dreft solution, but the absolute change is not great.

It is of interest that the initial, or cell, resistance accounts for 5 to 7% of the total resistance measured in the cake permeability runs, and numerically it is equivalent to 12 to 16% of the specific cake resistance although the units are different. It had been learned in previous work that the resistance of filter papers from the same box showed a considerable variation in resistance, sometimes as much as twofold even when micrometer-measured thicknesses were almost identical. It was for this reason that where possible blank-cell data were obtained with the same papers used as employed in the cake tests.

The cell-resistance values are somewhat greater, but by coincidence are roughly in line with the filtration rule-of-thumb approximation that R_m ordinarily is numerically equal to about one tenth of the specific resistance. However this approximation usually is applied to a filter cloth suited to the operation. But here the filter medium was a double acid-washed paper having very fast filtering characteristics, and that the approximation still holds is another indication of the unusually low resistance exhibited by the Lucite beds.

For cake-permeability run CC-2 cell-resistance values for the particular filter papers used were not available, and the data were calculated by means of R_m values from Figure 5.

Table 1. Specific Resistance of Lucite Spheres as Measured in a Compression-Permeability Cell

Run	Compressive stress, p_s, lb./sq. in.	Porosity, ϵ	Specific-cake resistance, α, (ft./lb.)(10^{-8})
CC-17	3.28	0.387	6.88
CC-17	15.78	0.384	7.02
CC-17	42.16	0.380	7.34
CC-17	55.80	0.379	7.48
CC-13	3.28	0.397	8.0
CC-13	15.78	0.386	8.04
CC-13	42.16	0.379	7.71
CC-13	55.80	0.377	7.86
CC-2	3.28	0.371	7.60
CC-2	15.78	0.370	7.86
CC-2	27.62	0.368	8.11
CC-2	46.07	0.364	8.65
CC-2	52.32	0.364	8.92

Comparison of Results with the Kozeny-Carman Equation

One of the most useful relationships so far derived for the flow of fluids through granular beds is the Kozeny-Carman equation:

$$u = \frac{dV}{Ad\theta} = \frac{\epsilon^3 \Delta P\, g_c}{k(1-\epsilon)^2 S_o^2 \mu L} \quad (6)$$

when one considers only the cake, and notes that

$$L = \frac{wV}{\rho_s (1-\epsilon) A} \quad (7)$$

the Kozeny-Carman equation and the filtration expression of Equation (1) can be combined to show that

$$\alpha = \frac{k(1-\epsilon)\, S_o^2}{\rho_S \epsilon^3} \quad (8)$$

Equation (8) predicts α to be linear in $[(1-\epsilon)/(\epsilon^3)]$ for a constant k and constant S_o. Any examination of permeability-cell data then should include a test of the relation between α and $[(1-\epsilon)/(\epsilon^3)]$. Figure 6 is a plot of corresponding values for the experimental points. A line expressing a relationship between the variables plotted must pass through the origin to be consistent with the fact that α is zero when $\epsilon = 1$. Therefore both the slope and the position are governed by the parameter $[(k S_o^2)/(\rho_s)]$. The data, which are admittedly meagre, align themselves fairly well except for the three points of run CC-13. Additional values are obviously needed before significant statements can be made. Further studies are underway; meanwhile the following are some tentative suggestions.

From the line drawn through the experimental data the value of $(k S_o^2)/(\rho_s)$ is 6.7 (10^7) ft./lb. The true solid density of Lucite was found to be 73.5 lb./cu. ft. When one takes this value and assumes $k = 5$, the specific surface turns out to be 31,000 sq. ft./cu. ft., or in metric units 1,019 sq. cm./cc. This value is 14.5% greater than the specific surface computed from the diameters microscopically measured. Another and perhaps more informative way of illustrating this discrepancy is to calculate the alphas predicted by Equation (8) when $k = 5$ and $S_o = 890$ sq. cm./cc. are used. This is shown by the lower line on Figure 6 which gives values for α which are 25 to 30% lower than the experimental alphas. The difference is too great to be accounted for by experimental error. The wall-effect correction suggested by Coulson (3) is minor in this case and reduces the difference by only 2%.

Now Carman suggested that $k=5 + 10\%$, and a larger value of k would reduce the difference. For spheres, though, recent published research (3, 9) confirms that k is close to 5 or perhaps slightly under. The specific surface from microscopic measurements also is believed to be correct. When this discrepancy was noted, additional microscopic counts were made with different sampling and counting techniques; the specific surface computed therefrom agreed well with the 890 sq. cm./cc. value.

The vacuum- and centrifugal-filtration results which follow give strong support to the experimental permeability data. The alphas from these other methods are in very close agreement with the permeability-cell alphas.

Carman (2) has written that the Kozeny-type of equation should be applied only to beds with reasonably uniform pore sizes. Yet in this investigation there was over a thirtyfold range in size of the particles, from 3.5 to 115 μ. The pore sizes no doubt varied widely, and perhaps it is this nonuniformity of pore structure that accounts for the disagreement between the measured surface and that calculated from the equation

$$k S_o^2/\rho_s = 6.7 \times 10^7$$

One can speculate also on the possibility that surface effects have influenced the experimental results. The addition of Dreft caused the Lucite spheres to be wetted and dispersed, but it did so by changing both the liquid-solid contact angle as well as the surface tension. Grace's work (4) has demonstrated the strong effect of electrolyte concentration on resistance and compressibility of filter cakes. Adsorption phenomena at the solid surface are no doubt involved, and research in this area might yield significant advances.

Vacuum-Filtration Results

The permeability-cell results discussed in the previous section constitute data on the point specific resistance of a porous bed under various compressive stresses. With the vacuum filter however it is the average specific resistance of a filter cake at a given ΔP that is measured in a constant-pressure filtration. Generally speaking therefore the point data must be integrated across the filter cake in order to arrive at the average resistance of the cake. But for an incompressible bed there is little or no variation in the point data, and the point and average specific resistances are the same. Since the cakes formed by the Lucite spheres are essentially incompressible, the specific resistances measured in the permeability cell and vacuum filter may be compared directly.

Filtration tests were made in the vacuum-filter cell in the manner described in the earlier section. Two types of runs were made: runs in which the slurry liquid was deaerated prior to filtration and runs in which the liquid was not degasified. While data from only the former runs are pertinent to the objective of this study, the effects of dissolved gases in the slurry of a vacuum filter are worthy of presentation.

The results of vacuum-filtration run F-7, presented in Figure 7, represent typical vacuum-filtration results for the deaerated slurries. The behavior of the system is in accord with filtration theory as shown by the straight-line plots of Figure 7. When θ/V is plotted against V for a constant ΔP filtration, a straight line should result whether the cake is compressible or incompressible.

For filtration runs F-3, F-4, F-5, and F-6 the water used in making up these slurries was not degasified prior to use. It was observed that the higher the vacuum applied, the greater the departure from linearity. Also the longer the filtration continues (the greater the volume of filtrate), the more pronounced the deviation becomes.

Since the plots of the deaerated slurries exhibit excellent linearity, these effects are believed due solely to the formation of gas or vapor within the cake. While runs F-3 through F-6 were in progress, large numbers of gas bubbles flowed in the filtrate delivery line, and the proportion of gas bubbles was greater at the higher vacua.

As the liquid enters and passes through the cake, it encounters regions of pressure progressively lower than atmospheric. Since the liquid was gas saturated or nearly so at atmospheric pressure, air or gas comes out of solution in these regions of lower pressure forming bubbles within the voids of the cake. Two-phase flow arises with both gas and liquid competing for the available flow channels. Effectively then the flow cross section for liquid is reduced. The effect is worse at higher vacua because more gas comes out of solution and also because at the lower absolute pressure the gas itself occupies a greater volume. In addition as the time of filtration lengthens, more and more of the flow channels would be expected to become vapor bound.

As pointed out earlier, the behavior of the deaerated slurries was in excellent agreement with theory, and the filtration constants were calculated from these runs. The concentration of the slurry was computed from the weight of the filter cake and the volume of filtrate collected. Well-used cotton-drill filter cloths were employed in these runs. The initial resistance and the average specific resistance computed from the intercept and slope appear in Table 2. The variation of values of α is less than 4.5%, and the maximum deviation from the mean value of 6.63 (10^8) ft./lb. is less than 2.3%. The initial resistance is numerically equal to about 36% of the value of α. All the filter cloths had been used in earlier filtrations of Lucite slurries, and very little change in initial resistance was exhibited.

Cake thickness was measured only for runs F-4, F-5, and F-6. Consequently information on cake porosity is available only on these three runs. The measurements and values of porosity calculated therefrom are given in Table 3.

The essential constancy of both the specific resistance and the porosity again confirm the incompressible nature of the cakes which was observed in the permeability tests.

Centrifugal-Filtration Results

From the preliminary experiments with the test centrifuge measurements were obtained to establish the relationship between the cake weight and thickness. A plot was made of dry-cake

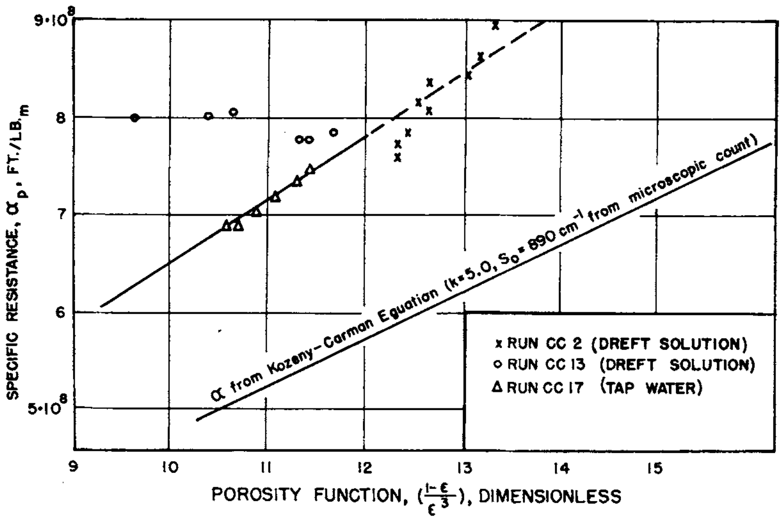

Fig. 6. Specific resistance as related to the porosity function of the Kozeny-Carman equation.

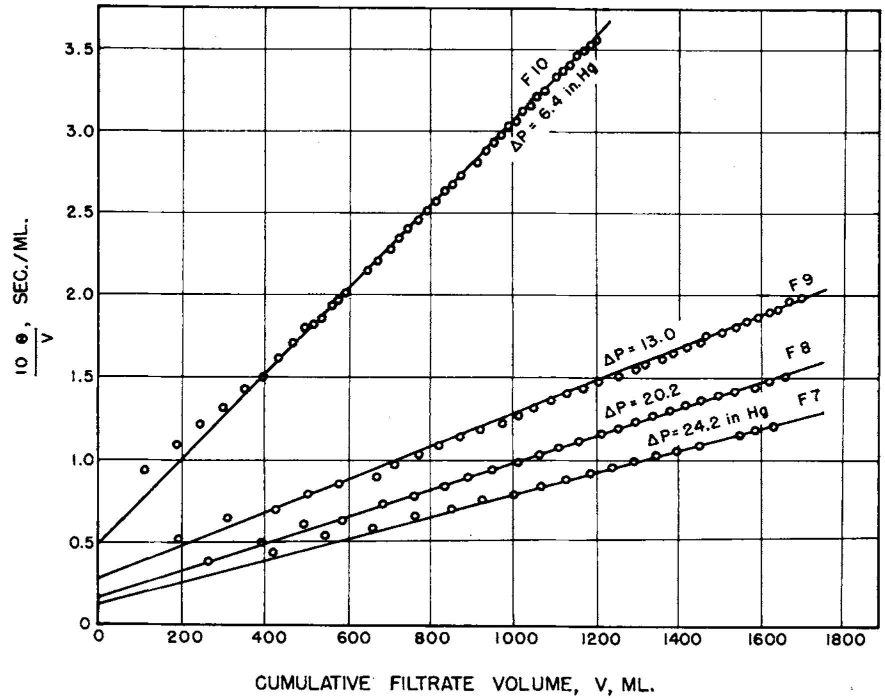

Fig. 7. Vacuum-filter data with degassed liquid used.

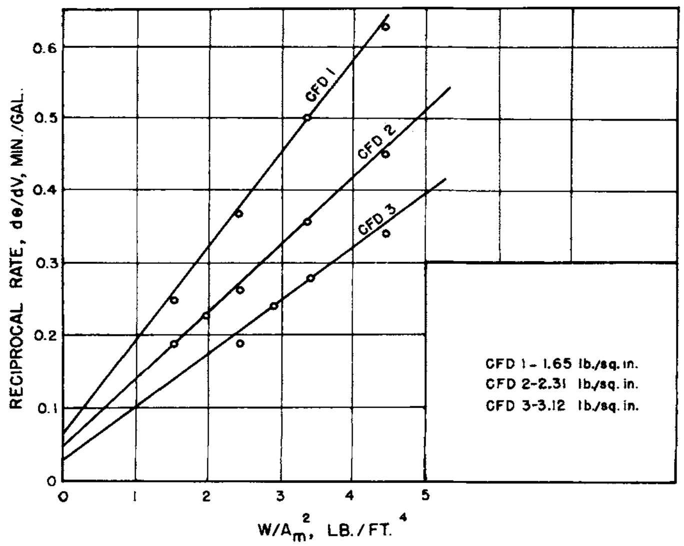

Fig. 8. Centrifugal-drainage rates under constant-pressure conditions.

TABLE 2. FILTRATION CONSTANTS CALCULATED FROM VACUUM-FILTRATION TESTS

Run	ΔP, in. Hg	α, ft./lb.	R_m, 1/ft.
F-7	24.2	6.48(10^8)	2.30(10^8)
F-8	20.2	6.52(10^8)	2.59(10^8)
F-9	13.04	6.76(10^8)	2.38(10^8)
F-10	6.37	6.76(10^8)	2.23(10^8)

weight against $(r_o^2 - r_c^2)$, the difference of the squares of the outer and inner radii respectively. The solids mass balance equation is

$$W = \pi\,(r_o^2 - r_c^2) h\,(1-\epsilon)\,\rho_s \quad (9)$$

Therefore from the slope of the line on the plot the porosity can be computed. The plot shows the line to be straight, thus implying a constant value for ϵ. In view of the earlier results this should be expected. The centrifuge cake porosity derived from the plot is 0.376.

When one allows for the initial resistance, the centrifugal-filtration rate equation, Equation (5), is written as

$$q = \frac{dV}{d\theta} = \frac{\Delta P\, g_c}{\mu\left[\dfrac{\alpha W}{A_m^2} + \dfrac{R_m}{A_o}\right]} = \frac{\frac{1}{2}(2\pi N)^2 \rho_L\,(r_o^2 - r_L^2)}{\mu\left[\dfrac{\alpha W}{A_m^2} + \dfrac{R_m}{A_o}\right]} \quad (10)$$

If this now be inverted to

$$\frac{d\theta}{dV} = \frac{\mu\,\alpha}{\Delta P\, g_c}\,\frac{W}{A_m^2} + \frac{\mu}{\Delta P\, g_c\, A_o} R_m \quad (11)$$

TABLE 3. CAKE POROSITIES MEASURED IN VACUUM-FILTRATION STUDY

Area of filtration (all runs) = 0.890 sq. in.

Run	ΔP, in. Hg	Weight of dry-cake solids, g.	Thickness, in.	Porosity
F-4	13.25	16.67	1.59	0.391
F-5	20.08	14.92	1.49	0.401
F-6	24.08	16.71	1.59	0.390
			Average	0.394

TABLE 4. FILTRATION CONSTANTS CALCULATED FROM CONSTANT-PRESSURE CENTRIFUGAL-DRAINAGE RATE STUDIES

Run	ΔP, lb./sq. in.	α, ft./lb.	R_m, 1/ft.
CFD-1	1.65	7.58(10^8)	4.39(10^8)
CFD-2	2.31	7.52(10^8)	4.88(10^8)
CFD-3	3.12	8.04(10^8)	4.49(10^8)

the equation shows that the reciprocal rate $d\theta/dV$ should plot as a straight line against W/A_m^2 for a constant-head centrifugal filtration and that α could be determined from the slope and R_m from the intercept.

Drainage-rate runs CFD-1, CFD-2, and CFD-3 each were made at a constant speed and constant liquid level. Flow rates were measured for various cake weights. These data therefore represent constant-head runs, and Figure 8 is a plot of the results. The arithmetic mean area was used as A_m^2 for the reasons advanced in the discussion of proposed centrifugal rate equations.

The relationship predicted by Equation (11) is confirmed and the values of α and R_m determined from the slope and intercept appear in Table 4.

The proportionality between the flow rate and the hydraulic pressure drop was checked by the plot of experimental data in Figure 9 and found to be validated.

The relationship between q and N, as well as q and $(r_o^2 - r_L^2)$ were tested by measuring corresponding values of drainage rate and liquid level, determined at various speeds for a 5-lb. Lucite filter cake. In Figure 11 $(r_o^2 - r_L^2)$ is plotted vs. $dV/d\theta$ on log-log coordinates, and Figure 10 is a log-log plot of N against $dV/d\theta$. The data points fit the theoretical relationship very well.

Although the system used in this study is so free filtering that R_m constitutes a sizable portion of the total resistance, there are many filter cakes for which R_m could be neglected. In these cases $d\theta/dV$ can be plotted against $\mu W/\Delta P g_c A_m^2$ to yield a straight line having a slope of α. Or, if $R_m/\Delta P$ were a constant, Equation (11) shows the data analysis still might be made by such a plot. These methods could not be expected to work however for compressible cakes or under any other conditions for which Equations (10) and (11) are invalid.

In the description of the centrifugal equipment and procedure reference

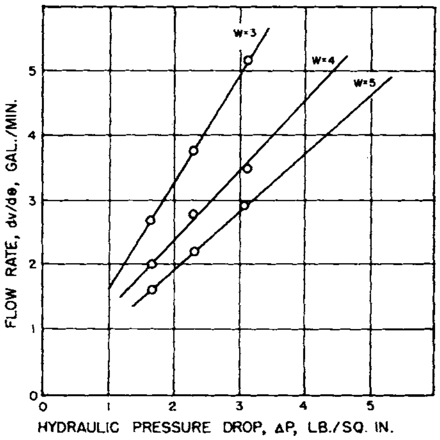

Fig. 9. Relationship between hydraulic pressure and flow rates in the centrifuge.

was made to a regular constant-pressure centrifugal filtration. Two attempts to obtain valid data in such runs were made during this investigation. The major obstacle to success in both trials was the impossibility of maintaining constant-head conditions over the entire portion of the run. At a specified rotational speed the restriction of constant hydraulic head requires a fixed liquid level. But because the cake must never become dry, the minimum height of liquid carried in the basket will be dictated by the level necessary to just cover the maximum thickness of cake.

In the first trial, designated as CF-1, a slurry of Lucite particles in Dreft solution was fed to the centrifuge. The rate of filtration was so great that the required liquid level could not be held until the run was almost over. Consequently, the valid data points were so few that no reliance could be placed on the constants derived from them.

In run CF-2 an attempt was made to overcome the fast filtration difficulties and still obtain valid measurements. This was done by making the filtrate a 65 wt. % glycerin-water solution which at the filtration temperature of 86°F. had a viscosity of 9.02 centipoises. The addition of 0.05% Dreft ensured the dispersion of the Lucite beads without affecting either the density or viscosity of the glycerin solution. The density of the solution was 1.15 g./ml., which being so close to the solid density of 1.18 minimized the settling tendency of the solid particles.

While these expediencies reduced the experimental difficulties, they did not eliminate them. The liquid height in the basket reached the prescribed level only after 130 sec.; therefore approximately the first 30% of the run could not be considered for the determination of the constants of the system.

Equation (11) can be formulated in terms of the mass rather than the volume of filtrate collected:

$$\frac{d\theta}{dM} = \frac{\mu \alpha x M}{\Delta P g_c \rho_L A_m^2} + \frac{\mu R_m}{\Delta P g_c \rho_L A_o} \tag{12}$$

where $dM = \rho_L dV$.

The results of run CF-2 are plotted in Figure 12 with increments in reciprocal rate, $\Delta\theta/\Delta M$ as the ordinate, and M/A_m^2 as the abscissa. Graphical treatment of this kind in which raw-data increments are plotted constitutes a very severe test of the data because it emphasizes irregularities in the individual measurements. As readings were taken in this run only 10 sec. apart, a small difference in timing represents relatively a much larger fraction of the time interval. Moreover an error in any interval is reflected in the following increment as well; for example if the filtrate reading for a given interval is taken slightly early and the succeeding reading is taken correctly, the first ΔM will be smaller and the second ΔM larger than their true values.

The scatter exhibited by the plot is more or less typical of experimental filtration data treated in this manner. The straight line through the latter portion of the data was drawn by eye and without prejudice to any one point

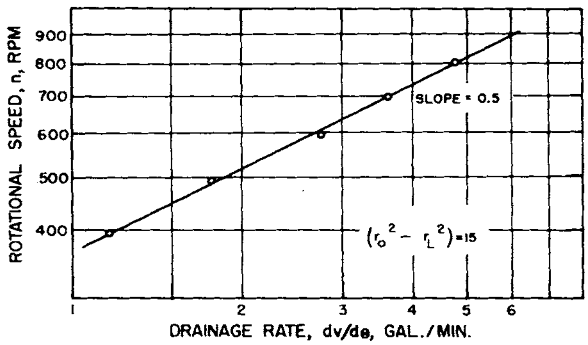

Fig. 10. Relationship between drainage rate and rotational speed.

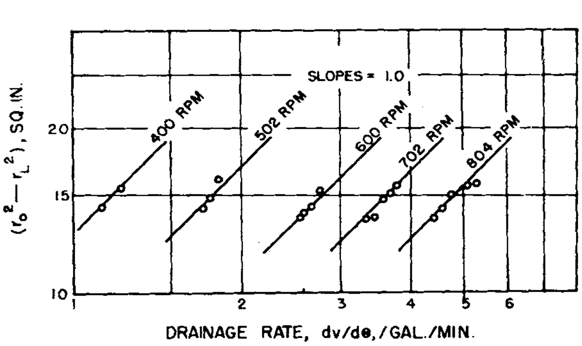

Fig. 11. Relationship between drainage rate and liquid-head function.

Table 5. Summary of Experimental Results for Comparative Purposes

Run	ΔP, lb./sq. in.	α, ft./lb.	R_m, 1/ft.
Vacuum filtration			
F-7	11.9	6.48(10^8)	2.30(10^8)
F-8	9.9	6.52(10^8)	2.59(10^8)
F-9	6.4	6.76(10^8)	2.38(10^8)
F-10	3.1	6.76(10^8)	2.23(10^8)
Average of runs		6.63(10^8)	2.38(10^8)
Centrifugal drainage rate			
CFD-1	1.65	7.58(10^8)	4.39(10^8)
CFD-2	2.31	7.52(10^8)	4.88(10^8)
CFD-3	3.12	8.04(10^8)	4.49(10^8)
Average of runs		7.71(10^8)	4.58(10^8)
Centrifugal filtration			
CF-2	4.24	9.2(10^8)	
Permeability cell General plot at			
$\epsilon = 0.376$		7.64(10^8)	
$\epsilon = 0.394$		6.43(10^8)	
Range of experimentally measured values		6.8(10^8) to 9.2(10^8)	

or group of points. From the slope of the line the value of α was determined by means of the experimentally measured properties of the system. This value of α was 9.2 (10^8) ft./lb.; it is about 19% greater than the average value of 7.71 (10^8) found in the drainage-rate runs. The agreement would be considered good in any case, but especially so in view of the conditions surrounding the runs under comparison.

COMPARISON OF PERMEABILITY-CELL, VACUUM-FILTRATION, AND CENTRIFUGAL-FILTRATION RESULTS

For comparative purposes the experimental results have been collected into a summary table, Table 5.

All values of α measured and reported in Table 5 are between 6.5 and 9.2 (10^8) ft./lb., which is a narrow range considering the methods and conditions employed. To examine the figures at approximately the same ΔP the data taken at about 3 lb./sq. in. were extracted and grouped together in Table 6. The correspondence in values is excellent, the total variation being less than 20%.

The Kozeny-Carman equation indicates the very great influence of porosity on specific resistance. A comparison of the vacuum-filtration and centrifugal-filtration alphas with the corresponding permeability-cell alphas then is of interest. Unfortunately porosity information of the vacuum-filtration tests F-7 through F-10 is lacking, but data are available on the cakes formed in runs F-4, F-5, and F-6, the ϵ's being 0.391, 0.401, and 0.390 respectively. These values do not differ much from one another, and it seems reasonable that the other vacuum-filtration cakes would not have a widely different porosity. Then if one assumes that the average porosity of 0.394 is representative of the vacuum-filtration cakes, the specific resistance of the filtration cakes can be compared to the permeability-cell value at this same porosity. Going through the α vs. $[(1-\epsilon)/(\epsilon^3)]$ plot one finds that the permeability-cell alpha at $\epsilon - 0.394$ is 6.43 (10^8), which is very close to the 6.63 (10^8) average for the filtration cakes.

Table 6. Comparison of Specific Resistances at the Same Pressure Drop Across the Cake

Method	Run	ΔP, lb./ sq. in.	α, ft./lb.
Vacuum filtration	F-10	3.1	6.76(10^8)
Centrifugal drainage	CFD-3	3.12	8.04(10^8)
Permeability cell	CC-13	3.28	*8.0(10^8)
	CC-17	3.28	**6.88(10^8)

* Dreft solution.
** Tap water.

The porosity of the cakes deposited in the centrifugal drainage rate studies was 0.376. At this porosity the specific resistance indicated by the permeability cell is 7.64 (10^8), whereas the average of the centrifugal drainage-rate tests was 7.71 (10^8).

There is thus remarkable agreement between the permeability-cell values and the other tests when compared at the same porosities. This suggests strongly that the specific resistance of an incompressible filter cake (whether vacuum or centrifugal) can be predicted with greater accuracy from a laboratory permeability test at the same porosity rather than at the same compressive stress. This probably is not so surprising since the porosity is the basic parameter.

The Kozeny-Carman equation shows that the specific resistance is completely determined by the porosity if k and S_o are constant. While the fraction voids of an unconsolidated bed are determined mainly by the compressive stress, it is an experimental fact that uniform solid particles in random packing can form different porosities under the same compressive loading. However the range of porosity values generally is not large. Vibration or tapping of a porous cake can cause rearrangement of particles into a tighter mode of packing, and in some circumstances the possibility of bridging exists. Of even greater importance to mixtures of different sizes is the matter of segregation, either in the slurry prior to deposition or while a layer is being deposited.

Even though the results of this work are encouraging, much remains to be done in the combined areas of filtration and centrifugation. Studies are needed to explain the discrepancy between the specific resistance determined by three types of measurement and that predicted by the Kozeny-Carman equation. The behavior of compressible materials needs more complete investigation. The performance of large-scale

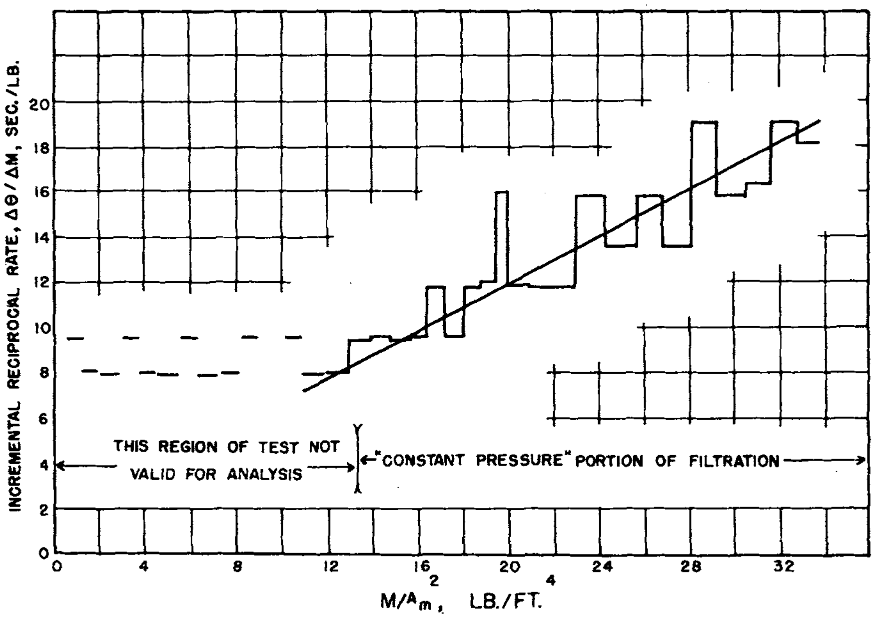

Fig. 12. Centrifugal-filtration data for calculating specific resistance.

centrifugal-filtering equipment needs to be compared with that of small-scale operations.

CONCLUSIONS

Three types of equipment, a permeability cell, a vacuum filter, and a centrifuge. were successfully used to measure the specific resistance of a bed of spherical particles. The specific resistances agreed among themselves within 20% at the same pressure drop and within 3% at the same cake porosity.

These results constitute the first known experimental data to be reported on the resistances offered by beds of incompressible spheres in all three methods of operation. It is only the third known study of its kind to be done on any sort of porous bed.

The Kozeny-Carman equation predicts specific resistances which are 25% lower than those measured in this work. The matter calls for further investigation.

ACKNOWLEDGMENT

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NOTATION

A = area, sq. ft. or sq. cm.
A_{avg} = arithmetic mean area, sq. ft. or sq. cm.
A_{lm} = logarithmic mean area, sq. ft. or sq. cm.
A_o = area of the filter cloth, sq. ft. or sq. cm.
c = concentration of solids in the slurry expressed as volume of cake forming solids per unit volume of filtrate, dimensionless
D = diameter of particle, ft. or cm.
g = local acceleration of gravity, ft./(sec.)(sec.)
g_c = dimensional constant
h = height of centrifugal basket in centrifugal filtration equation, ft. or cm.
k = Kozeny-Carman constant
K_s = permeability in Storrow equation, $K_s = (\rho_L g)/\alpha_v$
L = length of porous bed in direction of flow, ft. or cm.
M = mass of filtrate, lb. or g.
N = rotational speed, rev./sec.
ΔP = pressure driving force or pressure drop, lb./sq. in. or lb./sq. ft.
q = volumetric flow rate, cu. ft./sec. or cc./sec.
Q = steady state volumetric flow rate in the Storrow equation, cu. ft./sec. or cc./sec.
r_c = radius to inside or exposed surface of centrifuge cake, ft.
r_L = radius to liquid level, ft.
r_o = radius to outside surface of centrifuge cake, ft.
R_m = initial or medium resistance, 1/(ft.)
S_o = solids surface area per unit volume of bed, sq. cm./cc.
V = volume of filtrate, cu. ft. or cc.
w = concentration of solids in slurry expressed as mass of solids per unit volume of filtrate, lb./cu. ft. or g./cc.
W = mass of solids in bed or cake, lb. or g.
x = mass fraction of cake solids deposited per unit mass of filtrate

Greek Letters

α = average specific resistance on a mass basis, ft./lb.
α_v = average specific resistance on a volume basis, ft./cu. ft., $\alpha_v = (1-\epsilon)(\rho_s)$
ϵ = cake porosity or fraction voids, dimensionless
θ = time, sec.
ρ_L = density of liquid, lb./cu. ft. or g./cc.
ρ_s = true density of solid particles, lb./cu. ft or g./cc.
μ = fluid viscosity, lb./(ft.)(sec.)

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Pressure Drop and Liquid Holdup in Concurrent Gas Absorption

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This work presents initial information useful to concurrent gas-absorption design. Data are reported on the variation of pressure drop and liquid holdup obtained with various flow rates, packings, and liquids.

The advantages of concurrent operation for gas absorption were discussed earlier (*1*). Where applicable, this mode of operation may use uncommonly high flow rates, for which correlation and design information are unavailable. To supply such information on pressure drop and liquid holdup, the following work was performed.

PRESSURE DROP

With concurrent gas absorption the main determinant of operating condi-

Additional tables of data and literature references for contiguous countercurrent flow data, coded to type of packing, are filed with the American Documentary Institute, Photoduplication Service, Library of Congress, Washington 25, D. C., as document 6138. They may be obtained for $1.25 for photoprints or 35-mm. microfilm.